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A generalized interior point algorithm for convex objective functions has been developed and applied to various large scale problems.

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Final Technical Report
Robust Optimization of Large-Scale Systems: An Emerging New Technology
1 October 1991 to 28 February 1995

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Robert Vanderbei
Princeton University

This report describes the research under the support of the Air Force Grant (AFOSR-91-0359) to Princeton University.

The primary research of this project was to develop practical techniques that give rise to robust solutions to optimization problems. To achieve this goal, we have carried out research on three key topics: (1) the design of efficient algorithms for solving the resulting large scale optimization problems; (2) implementation strategies for several significant real-world applications; and (3) research on the modeling of the stochastic parameters via scenario generation. The results on each of these areas are given below.

1. *Algorithms for Solving Robust Optimization Models*

The constraints in the basic robust optimization problem possesses a structure similar to the constraints in stochastic programming problems. They can be depicted in two basic ways: a compact form, where the non-anticipativity conditions have been reduced to a minimum set, or in a split variable form where the non-anticipativity constraints are modeled explicitly. The latter structure seems to be more amenable to interior point algorithms.

There are two critical differences between robust optimization and the traditional stochastic linear programming model. First, robust optimization stresses risk aversion via nonlinear (convex) utility curves as opposed to assuming expected value as the desired objective. Thus, robust optimization requires nonlinear programming solvers for its solution. Second, robust optimization does not require that the constraint set for the second and subsequent stages be feasible for all values of the first stage variables. We assume that the violation of these constraints is handled through a nonlinear penalty function. These issues are described in the report by Mulvey, Vanderbei and Zenios [MV2].

Since robust optimization requires nonlinear objectives, we focused on algorithms for solving the subsequent NLP problem (with linear constraints). Considerable progress has been made in this regard. We have improved the restarting techniques at the beginning of each iteration of our DQA algorithm, thus reducing total execution time by over 40% [M15]. More importantly, we have been able to achieve a higher degree of accuracy.

In addition, we extended the solution strategy in the interior point algorithm of Bob Vanderbei -- the LOQO solver [V11] -- to handle general convex objectives (along with quadratic objectives). This result is significant since the execution time to solve the general convex problem is only slightly longer than for solving linear programs. We have shown that robust optimization problems are therefore no more difficult to solve than stochastic linear programs. References [M2, M4, M6, M12, M14, M20, M21, M22, M23, V7, V9] describes these research results in more detail.

Recently, we specialized the factorization routine for the structure inherent in robust optimization models (the split variable formulation). See [MV1] for details of the computational tests. This approach is considerably more efficient than the traditional multiple min-degree ordering routine (over 50 times faster).
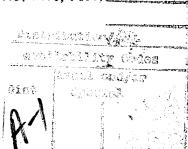
Our computational testing were conducted in several locations. First, we employed a network of UNIX (SGI) workstations at Princeton University. Next, we traveled to several supercomputer companies in order to test the viability of LOQO and the DQA algorithm in parallel computer environments. This work was conducted at Convex Corp. in Dallas, Texas, at Thinking Machines in Boston, MA and at Silicon Graphics in Mountain View, CA. We showed that the very large robust optimization problems can be solved with both parallel and distributed computer architectures. Our larger test cases possess over 1,000,000 constraints and 1,700,000 nonlinear variables. These problems are among the largest nonlinear programs ever solved.

2. Strategies for Implementing Robust Optimization Models

We demonstrated that robust optimization can be used to efficiently model decision problems. We contacted several groups developing planning models, including the Air Force's Air Mobility Command (AMC) at Scott AFB, Bellcore in New Jersey, US West in Boulder Colorado, and several financial organizations. For example, we have worked with the AMC in order to take their flight planning model (STORM) and add stochastic scenarios to the problem. We developed a system for generating scenarios based on historical data on the demands for cargo shipments and used this system to generate the robust optimization problems. This robust STORM system has been sent to a number of stochastic programming researchers, including Gerd Infanger and George Dantzig at Stanford, S. Sen and Julia Higle at University of Arizona, Roger Wets at UC Davis, and Alan King at IBM research in Yorktown Heights, NY. We solved the STORM model with over 200 scenarios using our modest network of workstations at Princeton University. (See references [M2, M6, M14, M15, M19].)

In addition, we developed a telecommunication planning model that is robust with respect to network survivability. The goal is to design the network in a manner such that the projected calls can be routed, even if part of the network in brought down. The test case was worked out with researchers at Bellcore. Our design goal for this problem is to ensure that any one node could be dropped without adversely impacting the rest of the network. However, other design goals could be handled in the context of additional scenarios. This test case has been the focus of both modeling and algorithmic research. (See references [M7, M8].)

Finally, we developed some multi-stage planning systems for financial planning, in particular for integrating assets and liabilities. These problems are important for large institutions attempting to coordinating their cashflows over extended periods. The use of these integrative risk management models should reduce risks to large organizations. For example, an application would be the building and costing of aircraft over time. The model includes multiple periods out to 40 years. These models are discussed in references [M1, M3, M5, M9, M10, M13, M16, M17, M18].

3. *Modeling Stochastic Parameters and Scenario Generation*

In these robust optimization models, an important consideration is the handling of uncertainty -- for example, interest rates and inflation. The cost estimates will depend upon inflation and hence the liabilities must be linked to inflation as determined by the scenarios. A coherent picture is needed. We have developed a scenario generation procedure for the economic factors which is consistent with respect to academic research and is easily incorporated in a robust optimization framework. Several meetings have been conducted with David Gay (AT&T) and Robert Fourer (Northwestern University) in order to extend the AMPL modeling language to automatically handle scenario generation. In addition, we have met several people at Frontline systems to show them how to include robust optimization in their spreadsheet packages. Frontline has implemented robust optimization in Excel and other major spreadsheet vendors may do the same. At present, these systems are available as add-in routines to the spreadsheets. A goal of this effort was to provide interfaces that are convenient and easily managed, so that robust optimization can be performed without a major investment in time or effort. This would improve the chances of using robust optimization in practice.

Next, we investigated the need for multi-objective methods in robust optimization. It should be recalled that robustness can be achieved but there might be additional costs. The cost depends upon the actual circumstances of the application. Since robust optimization can be used for long term planning, we need to consider preferences over time. See reference [M11] for a preliminary discussion of the issues that are pertinent for individuals when planning their financial future.

Major Research Accomplishments during Grant

1. *Generalized interior point algorithms to handle convex objectives and demonstrated that approach is efficient for large scale problems.*

An important consideration for robust optimization is modeling risk aversion with nonlinear (convex) objective functions. The most important objective is to maximize expected utility over the range of possible scenarios. Including nonlinear objectives in the interior point algorithms has been quite important. We showed that solving the robust optimization problems with convex objective is only slightly more difficult to solve than solving the traditional stochastic linear programs. Thus, it is likely that nonlinear objectives will not be a barrier to the use of robust optimization.

2. *Implemented the LOQO and DQA systems using several computer hardware architectures, including parallel and distributed computers, and increased the size of solvable robust optimization problems.*

Robust optimization problems grow quickly as a function of the number of scenarios. However, we like to include a wide range of possible scenarios to model the stochastic parameters in a realistic fashion. Thus, we are faced the tradeoff of numerical efficiency (or even solvability) versus modeling accuracy. It is therefore important for us to have the ability to solve problems with a moderate number of scenarios (at least several hundred to several thousand). We demonstrated that the DQA method is highly effective in this environment. More recently, we showed that direct solvers can also be implemented on parallel-processing computers by parallelizing the factorization routine. This result might apply to the more general area of large scale optimization, but more research is needed to determine the degree of generality of the results.

3. *Specialized interior point codes.*

We developed a preordering routine that greatly reduces the time to factor the (ADA') matrix -- a key step in the interior point codes. The preordering is called *tree dissection* and takes advantage of the structure of the split-variable robust optimization model. See reference [MV1] for a discussion of the preliminary tests of the concepts.

4. *Developed several real-world models of robust optimization.*

Robust optimization aims to be a practical approach for solving planning problems. We demonstrated the value of robust optimization by developing and solving planning models for real problems. Several models have been built, including a robust STORM, a telecommunication survivability model, and several multi-stage financial planning models. Included in this work is the generation of the scenarios to represent the range of likely uncertainties.

Publications

John M. Mulvey

M1   "Asset and Liability Allocation in a Global Environment," Princeton University Report SOR-95-3, to appear in *Finance*. North Holland Publishing Company, 1996 (with W. Ziemba).

M2   "A Distributed Scenario Decomposition Algorithm for Large Stochastic Programs," Princeton University Report SOR-93-2, to appear in *ORSA Journal on Computing*, 1995.

M3   "Financial Planning via Multi-Stage Stochastic Optimization," Princeton University Report SOR-94-09, to appear as a feature article in *ORSA Journal on Computing*, 1995.

M4   "Solving Dynamic Stochastic Control Problems in Finance Using Tabu Search with Variable Scaling," Princeton University Report SOR-94-13, to appear in *Proceedings of the Meta-Heuristics International Conference MIC-95*, Kluwer Academic Publishers, 1995 (with F. Glover and K. Hoyland).

M5   "Solving Global Optimization Problems in Long-Term Financial Planning," Princeton University Report SOR 95-01, (with A. Berger and F. Glover), 1995.

M6   "A New Scenario Decomposition Method for Large-Scale Stochastic Optimization," *Operations Research*, **43**, 3, 1995, 477-490 (with A. Ruszczynski).

M7   "Making the Case for Robust Optimization Models," Princeton University Report SOR-94-17, 1994, (with D. Bai and T. Carpenter), 1994.

M8   "Stochastic Programming Models for Network Survivability," Princeton University Report SOR-94-14, 1994 (with D. Bai and T. Carpenter).

M9   "Financial Planning via Multi-Stage Stochastic Programs," in Proceedings of the 15th International Symposium on Mathematical Programming, University of Michigan, 1994.

M10 "Stochastic Programming in Industrial Engineering," Princeton University Report SOR-94-12, 1994 (with J. Birge), to appear in *Mathematical Programming for Industrial Engineers*, M. Avriel and B. Golany (eds.), forthcoming.

M11 "Errors in Asset Management," Princeton University Report SOR-94-08, 1994 (with A. Berger).

M12 "Solving Stochastic Control Problems in Finance via Global Optimization," Princeton University Report SOR-94-01, 1994 (with C. Maranas, I. Androulakis, C. Floudas, and A. Berger).

M13 "Capturing the Correlations of Fixed-Income Instruments," *Management Science*, **40**, 10, 1994, 1329-1342 (with S. Zenios).

M14 "An Extension of the DQA algorithm to convex stochastic programs," *SIAM Journal on Optimization*, 4, 4, 1994, 735-753 (with A. Berger and A. Ruszczynski).

M15 "Restarting Strategies for the DQA Algorithm," in *Large Scale Optimization: State of the Art*, Kluwer Academic Publishers, 1994, 1-25 (with A. Berger and A. Ruszczynski).

M16 "Multi-Stage Financial Planning Systems", in *Operations Research Models in Quantitative Finance*, R. D'Ecclesia and S. Zenios (eds.), 1994.

M17 "Dynamic Diversification of Fixed Income Portfolios," *Financial Analysis Journal*, January - February 1994, 30-38 (with S. Zenios).

M18 "Integrating Assets and Liabilities for Large Financial Organizations," Chapter in *New Directions in Computational Economics*, W. W. Cooper and A. B. Whinston (eds.), Kluwer Academic Publishers, 1994, 135-150.

M19 "Robust Optimization on PC Supercomputer," SIAM Newsletter, November 1993.

M20 "Higher Order Predictor-Corrector Interior Point Methods with Application to Quadratic Objectives," *SIAM Journal on Optimization*, 3, 4, 1993, 696-725 (with T. Carpenter, I. Lustig, and D. Shanno).

M21 "Separable Quadratic Programming Via Primal-Dual Interior Point Method and Its Use in a Sequential Procedure," *ORSA Journal of Computing*, 5, 2, Spring 1993, 182-191 (with T. Carpenter, I. Lustig and D. Shanno).

M22 "A D.Q.A. Method for Multistage Stochastic Program Problems," IFIP Conference on System Modelling and Optimization, Zurich, Switzerland, *Lecture Notes on Control and Optimization*, 1992 (with A. Ruszczynski).

M23 "A Diagonal Quadratic Approximation Method for Large Scale Linear Programs," *OR Letters*, 12, 1992, 205-215 (with A. Ruszczynski).

Robert J. Vanderbei

V1 "Symmetric Quasi-definite Matrices," *SIAM Journal on Optimization*, to appear 1995.

V2 "Affine-Scaling Trajectories Associated with a Semi-infinite Linear Program," *Mathematics of Operations Research*, to appear 1995.

V3 "The Simplest Semidefinite Programs are Trivial," *Mathematics of Operations Research*, to appear 1995 (with B. Yang).

V4 "The Complex Zeros of Random Polynomials," *Transactions of the AMS*, to appear 1995 (with L. Shepp).

V5 "A Probabilistic Formula for the Concave Hull of a Function," *Annals of Probability*, to appear 1995.

V6   "Interior-point Methods for Max-Min Eigenvalue Problems," *Mathematical Programming*, to appear 1995 (with F. Rendl and H. Wolkowicz).

V7   "Interior-point Methods: Algorithms and Formulations," *ORSA Journal on Computing*, 6, 1994, 32-34.

V8   "Brownian Bandits," in *The Dynkin Festschrift*, M. Freidlin (ed.), 1994.

V9   "An Algorithmic and Numerical Comparison of Several Interior-point Algorithms," Princeton University Report SOR-95-05, 1994 (with A. Duarte and B. Kang).

V10   "An EM approach to OD Matrix Estimation," Princeton University Report SOR-94-04, 1994 (with J. Iannone).

V11   "LOQO: An Interior point Code for Quadratic Programming," Princeton University Report SOR-94-15, 1994.

V12   "Two-Thirds is Sharp for Affine Scaling," *OR Letters*, 13, 197-201, 1993 (with L. Hall).

John M. Mulvey and Robert J. Vanderbei

MV1 "Solving Multistage Stochastic Programs using Tree Dissection," Princeton University Report SOR 95-07, (with A. Berger and E. Rothberg), 1995.

MV2 "Robust Optimization of Large-Scale Systems," *Operations Research*, 43, 2, 1995, 264-281 (with S. Zenios).

Presentation of Research Results

The principle investigators have presented the research results at many professional meetings around the world. Also, lectures have been given at Air Force sponsored meetings such as "Joint Operation Research/Artificial Intelligence Workshop on Transportation Planning" at Carnegie Mellon University, Pittsburgh, Pennsylvania, on September 30 - October 2, 1991, as well as several conferences sponsored by the Air Force at Scott AFB. Approximately 30-35 lectures were given on the topic of robust optimization and related solution algorithms during the project tenure.